

Ramakrishna Mission Residential College (Autonomous)
Affiliated to the University of Calcutta
Narendrapur, Kolkata 700103, West Bengal, India

UG Admission Test, 2017
Subject : Mathematics Honours

Full Marks : 50

Duration : 1 hour 30 minutes

Group—A

(Answer all the questions, each carrying 2 marks)

1. If $(1 + i)(1 + 2i) \dots \dots (1 + ni) = x + iy$, then find the value of $2.5.10 \dots \dots (1 + n^2)$ where 'i' is the imaginary unit.

2. Let $f(x) = \begin{cases} \frac{\log(1+\sin 2x)}{x} & ; x \neq 0 \\ 1+a & ; x = 0 \end{cases}$

Find the value of 'a' for which $f(x)$ is continuous at $x = 0$.

3. Let f be a positive function and $I_1 = \int_{1-k}^k xf(x(1-x))dx$ and $I_2 = \int_{1-k}^k f(x(1-x))dx$ where $(2k - 1) > 0$. Then find the value of $\frac{I_1}{I_2}$.

4. If A is a skew symmetric matrix of order 3 and P be a 3×1 matrix, prove that $P^TAP = O$; P^T is the transpose of P .

5. Test whether the function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = [x]$, $\forall x \in \mathbb{R}$ is bijective or not. Justify your answer.

6. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then find the value of $\lim_{x \rightarrow a} \frac{g(x)f(a) - g(a)f(x)}{x-a}$.

7. Find the differential equation of the family of circles, each having its centre on the straight line $y = x$ and passing through the origin.

8. From an urn containing $2n$ balls, any number of balls are drawn. Then find the probability of drawing an even number of balls.

9. Find the remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9.

10. If \vec{a} , \vec{b} , \vec{c} are non-coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{\sqrt{2}}(\vec{b} + \vec{c})$, then find the angle between \vec{a} and \vec{b} .

11. A line segment of constant length $2a$ meets the x -axis at A and y -axis at B . If the line segment slides with its extremities on the co-ordinate axes, then find the equation of locus of the midpoint of the line segment.

12. Find the area of the region bounded by the curves $y = |x-1|$ and $y = 3-|x|$

13. For what value of n , $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ ($a \neq b$) is the A.M. of a and b ?

Group—B

(Answer all the questions, each carrying 3 marks)

14. Show that between two consecutive roots of the equation $e^x \sin x + 1 = 0$, there exists a root of the equation $\tan x + 1 = 0$.

15. What fraction exceeds its p -th power by the greatest number possible ?

16. Find the image of the point $(1, 2, -1)$ with respect to the plane $3x-5y+4z = 5$.

17. Find the number of real solutions of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$.

Group—C

(Answer all the questions, each carrying 4 marks)

18. How many arrangements can be formed by taking 4 letters at a time out of the letters of the word 'MATHEMATICS' ?

19. Without expanding a determinant at any stage, show that

$$\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = xA + B,$$

where A and B are determinants of order 3 not involving x .

20. Solve graphically, the following LPP (without using graph paper)

Minimize, $z = 3x_1 - 2x_2$

subject to $3x_1 + 4x_2 \geq 12$

$$x_1 - 3x_2 \leq 6$$

$$x_1 - 2x_2 \leq -4$$

$$x_1, x_2 \geq 0$$

(Rough Work)