Ramakrishna Mission Residential College (Autonomous)
Affiliated to the University of Calcutta
Narendrapur, Kolkata 700103, West Bengal, India

UG Admission Test, 2017
Subject : Statistics

Full Marks : 50 **Duration : 1 hour 30 minutes**

Group—A

(Multiple choice Answer type question of 2 marks each)
Rough work must be shown on the space provided. 2×15=30

1. Let $f : [0, 13] \to R$ be defined by $f(x) = x^{13} - e^{-x} + 5x + 6$. Then the minimum value of f on [0, 13] is at
   i) $x = 0$ ii) $x = 13$ iii) $x = \frac{6}{5}$ iv) $x = e$

2. The domain of definition of $f(x) = \log_e (x^2 - 2x - 3)$ is
   i) $(0, \infty)$ ii) $(-\infty, -1)$ iii) $(-\infty, -1) \cup (3, \infty)$ iv) $(-\infty, -3) \cup (1, \infty)$

3. Ten players are to play in a tennis tournament. The number of pairings for the first round is
   i) $\frac{10!}{2^5 5!}$ ii) 2^{10} iii) $\binom{10}{2}$ iv) ${}^{10}P_2$

4. Let x and y be two real numbers such that $2 \log_e (x-2y) = \log x + \log y$, holds. Which of the following is a possible value of $\frac{x}{y}$?
   i) 2 ii) 3 iii) 4 iv) 0

5. The value of the expression $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{99}+\sqrt{100}}$ is
   i) a rational number in (0, 9) ii) an irrational number in (0, 9)
   iii) a rational number in (0, 10) iv) an irrational number in (0, 10)

6. The maximum value of $f(x) = x^n (1-x)^n$ for natural number $n \geq 1$ and $0 \leq x \leq 1$ is
   i) $\frac{1}{2^n}$ ii) $\frac{1}{3^n}$ iii) $\frac{1}{4^n}$ iv) $\frac{1}{5^n}$

7. For what values of the real number 'a' does the quadratic equation : $x^2 + ax + a = 1$ have distinct real roots ?
   i) $a \neq 2$ ii) $a > 2$ iii) $a = 2$ iv) all values of a.

8. There are positive real numbers x and y which solve the equations : $2x + ky = 4$, $x + y = k$ for i) all values of k ii) no values of k iii) k = 2 only iv) only k > − 2.

9. The smallest value of $I(a) = \int_0^1 (x^2 - a)^2 dx$, as '$a$' varies is

i) $\frac{3}{20}$ ii) $\frac{4}{45}$ iii) $\frac{7}{13}$ iv) 1

10. The smallest positive integer n such that $1-2+3-4+5-6+\ldots+(-1)^{n+1} \cdot n \geq 100$ is

i) 99 ii) 101 iii) 199 iv) 300

11. The area of the region enclosed by the curve $y = x^2$ and the straight line $x+y = 2$ is

i) 3 ii) $\frac{27}{2}$ iii) $\frac{9}{2}$ iv) 9

12. A permutation of 1, 2,…100 is chosen at random then the probability that the numbers 1 and 100 appear next to each other equals

i) $\frac{1}{100}$ ii) $\frac{1}{50}$ iii) $\frac{1}{99}$ iv) $\frac{1}{98}$

13. The number of distinct real values of x for which the matrix $A = \begin{pmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{pmatrix}$ is singular is

i) 1 ii) 2 iii) 3 iv) 4

14. Let A(t) denote the area bounded by the carve $y=e^{-|x|}$, the x−axis and the straight lines $x = -t$ and $x = t$. Then $\lim_{t \to \infty} A(t)$ is equal to

i) 2 ii) 1 iii) $\frac{1}{2}$ iv) 0

15. The value of $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2 + kn}}$ is

i) $2(\sqrt{2}-1)$ ii) $2\sqrt{2}-1$ iii) $2-\sqrt{2}$ iv) $\frac{1}{2}(\sqrt{2}-1)$

Group—B

(Broad answer type question of 4 marks each)

Answer the questions on the spaces provided

4×5=20

16. Evaluate the determinant

$$\begin{vmatrix} 1+x_1y_1 & 1+x_1y_2 & 1+x_1y_3 \\ 1+x_2y_1 & 1+x_2y_2 & 1+x_2y_3 \\ 1+x_3y_1 & 1+x_3y_2 & 1+x_3y_3 \end{vmatrix}$$

17. a) For $0<x<1$, evaluate the infinite series $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots$

b) Determine the area enclosed by the curve : $|x| + |y| = 1$

18. a) It is given that $e^a+e^b = 10$. Then find the maximum value of $(e^a+e^b+e^{a+b}+1)$

b) Evaluate : $\int_{-2}^{4} [x]dx,$ where $[x]$ is the largest integer less than or equal to x.

19. Consider the function $f : R^+ \to R^+$, defined as $f(x) = \max\{x, x^2\}$, for all $x \in R^+ = (0, \infty)$.
a) Plot $f(x)$ b) Find a point where f is not differentiable and argue why it is not differentiable.

20. a) For $n \geq 1$, let g_n = the geometric mean of $\left\{\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}\right\}$. Find $\lim_{n\to\infty} g_n$

b) Find the maximum value of (xyz) subject to $x^2+2y^2+9z^2 = 6$.

(Rough Work)